Simultaneous design of topology and printing direction of structural elements for Wire-and-Arc Additive Manufacturing (WAAM)

M. Bruggi [1], V. Laghi [2], M. Palermo [2], T. Trombetti [2]

[1] *Department of Civil and Environmental Engineering, Politecnico di Milano, 20133 Milano, Italy, matteo.bruggi@polimi.it*

[2] *Department of Civil, Chemical, Environmental and Materials Engineering, University of Bologna, 40136 Bologna, Italy, vittoria.laghi2@unibo.it, michele.palermo7@unibo.it, tomaso.trombetti@unibo.it*

1. Introduction

Wire-and-Arc Additive Manufacturing (WAAM) is a metal 3D printing technique that allows creating innovative structural shapes [1]. As shown by few experimental investigations, the layer-by-layer manufacturing is responsible for remarkable anisotropy in the elastic response of the WAAM-produced material, see e.g. [2,3]. Topology optimization by distribution of isotropic material is a design tool that is extensively used to sketch lightweight structural components [4]. In this contribution, a suitable topology optimization technique is implemented to generate lightweight structures taking into full account the peculiar anisotropy of the WAAM alloy.

An orthotropic material model is reviewed, as derived from the data of an experimental investigation that was recently performed on printed alloys made with 308LSi stainless steel wire feed. Hence, a displacement-constrained minimum weight optimization procedure is implemented that exploits, as design variables, not only the density field of an orthotropic material phase, but also the orientation of the symmetry axes of such material with respect to a reference frame (i.e., the printing orientation used to manufacture the whole structural element).

The lightweight design of a cantilever beam is attacked to find optimal WAAM solutions that are compared to those achieved by performing topology optimization with isotropic stainless steel. The proposed numerical simulations assess that the printing direction remarkably affects the stiffness of the optimal layouts as well as its topology. Ongoing developments are also introduced to endow the proposed multi-constrained formulation with other structural requirements in the form of additional enforcements.

2. The optimization problem

A material characterization may be performed starting from suitable experimental tests performed on the printed WAAM plates [2,5,6]. The material symmetries of the plates created through the WAAM technique, see Figure 1, suggest the adoption of an orthotropic plane stress model. In the referenced figure, symmetry axes of the material are denoted as $\hat{x}_1$ and $\hat{x}_2$. The former lies along the printing direction, also labelled as the longitudinal direction L, whereas the latter is the transversal direction T.

By testing a dog-bone specimen along L, the relevant Young's modulus E_L and Poisson's ratio ν_{LT} may be derived, whereas E_T and ν_{TL} need for testing along the transversal direction T. Experimental evidence confirms that the following relation holds:

$$E_L \cdot \nu_{TL} = E_T \cdot \nu_{LT}. \tag{1}$$

Hence, by gathering the two-dimensional strain and stress components in vectors (Voigt's notation), the compliance matrix of a WAAM alloy may be written as:

$$\begin{Bmatrix} \varepsilon_L \\ \varepsilon_T \\ \gamma_{LT} \end{Bmatrix} = \begin{bmatrix} 1/E_L & -\nu_{TL}/E_T & 0 \\ -\nu_{LT}/E_L & 1/E_T & 0 \\ 0 & 0 & 1/G_{LT} \end{bmatrix} \begin{Bmatrix} \sigma_L \\ \sigma_T \\ \tau_{LT} \end{Bmatrix}, \tag{2}$$

where:

$$1/G_{LT} = 4/E_D - (1-\nu_{LT})/E_L - (1-\nu_{TL})/E_T, \tag{3}$$

being E_D the apparent value of the Young's modulus measured along the diagonal direction D, see [7]. This direction bisects the angle between the longitudinal direction and the transversal one. The average values from the experimental tests read: $E_L = 136MPa,\ E_T = 106MPa, \nu_{LT} = 0.47, \nu_{TL} = 0.37$, whereas the shear modulus $G_{LT} = 151MPa$. For the conventional Grade 304L isotropic steel, one has $E = 200MPa, \nu = 0.3$.

The design of two-dimensional structural elements for WAAM is herein formulated as a displacement-constrained minimum weight problem by distribution of orthotropic material [8]. A finite element discretization of a given design domain is needed, adopting four-node displacement-based elements and a set of element-wise discrete design variables. In the *i*-th of the *N* elements of the mesh, $0 < \rho_{\mathrm{i}} \leq 1$ is the minimization unknown that governs the "density" of the orthotropic material. Additionally, the variable $0 < \theta \leq 180°$, see Figure 1, governs the orientation of the printed layers. It is herein assumed that the printing direction does not change during the fabrication process, that means this variable takes the same value throughout the design domain. It is remarked that θ controls the (anticlockwise) rotation of the axis x_1 of the general reference system with respect to the axis $\hat{x}_1$ of the material reference system. Assuming that the design domain is framed within the general reference system, the printed direction with respect to the axis x_1 is that given by a (anticlockwise) rotation of this axis as per Figure 1.

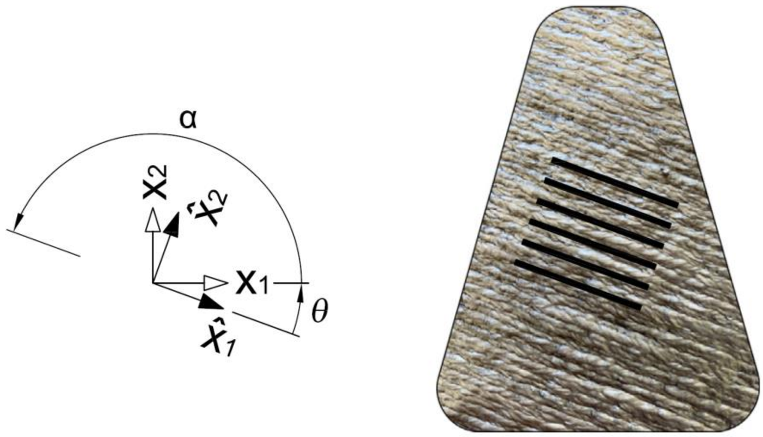

Figure 1. Orientation of the printed layers for an object framed in the general reference system with axes $x_1$and x_2. $\hat{x}_1$ and $\hat{x}_2$ are the symmetry axes of the alloy, i.e., the longitudinal direction (L) and the transversal direction (T), respectively.

The conventional Solid Isotropic Material with Penalization (SIMP) [9] is modified to handle the orthotropic allow as follows:

$$\boldsymbol{C}^{-1}(\rho_i, \theta) = \rho_i^p \; \boldsymbol{T}_\sigma(\theta)\, \widehat{\boldsymbol{C}}_{w,0}^{-1}\, \boldsymbol{T}_\sigma^T(\theta), \tag{4}$$

where $\boldsymbol{C}(\rho_i, \theta)$ is the compliance matrix of the *i*-th element in the general reference system, $\widehat{\boldsymbol{C}}_{w,0}$is that given in Eqn. (2), $\mathbf{T}_\sigma$ is a suitable transformation matrix and *p=3* is a penalization parameter for intermediate values of the density, see [8].

A problem for the simultaneous design of the topology and the printing direction of the WAAM alloy can be stated as:

$$\begin{cases} \min\limits_{0<\rho_i\leq 1, 0<\theta\leq 180°} W = \sum\limits_{i=1}^{N} \rho_i V_{0,i} \\ subject\ to \left(\sum\limits_{i=1}^{N} \rho_i^p\, \boldsymbol{K}_{0,i}(\theta)\right)\boldsymbol{U} = \boldsymbol{F} \\ u_a \leq u_{lim} \end{cases}. \tag{5}$$

In the above statement W is the objective function, i.e., the weight of the overall structure, which depends on the volume of each finite element scaled by the relevant "density" minimization unknown. The first constraint prescribes the discrete equilibrium of the structural element. The global stiffness matrix is found by assembling the element contributions that include the constitutive law given in Eqn. (4). The load vector $\boldsymbol{F}$ allows computing the nodal displacement vector $\boldsymbol{U}$ under the effect of a given load. In this contribution an example based on a single point force (vertical load) is considered. The deflection at the load application point u_a is the controlled displacement, whereas u_{lim} is the value of the maximum allowed displacement in that location. The problem is solved using sequential convex programming [10]. The adjoint method is used to compute the sensitivity of the objective function w.r.t. to the minimization unknowns; a standard linear filter is used for the densities to avoid numerical instabilities [9].

3. Design example

A rectangular domain is considered, having width 80cm and height 40cm, as shown in Figure 2. The thickness of the printed lamina is 4mm. The specimen is subjected to a vertical force F=8.33kN (per mm of thickness of the WAAM-printed plate), which is located at the midpoint of the right side. The left side is fully clamped. The allowed displacement u_{lim} is equal to 4mm.

Figure 3 reports the two mirrored solutions that have been found depending on the initial guess used to start the optimization.

In both cases, the weight at convergence is around 40% (ratio of the black region to the rectangular design domain), against the value 36% that is needed in case of the conventional Grade 304L isotropic steel. A lack of symmetry affects the results involving the WAAM alloy. Indeed, the orientation of the printed material is such that the elastic modulus along any pair of symmetric directions with respect to x_1 (or x_2) takes values that can be very different from each other, see Figure 4.

Figure 2. A cantilever beam.

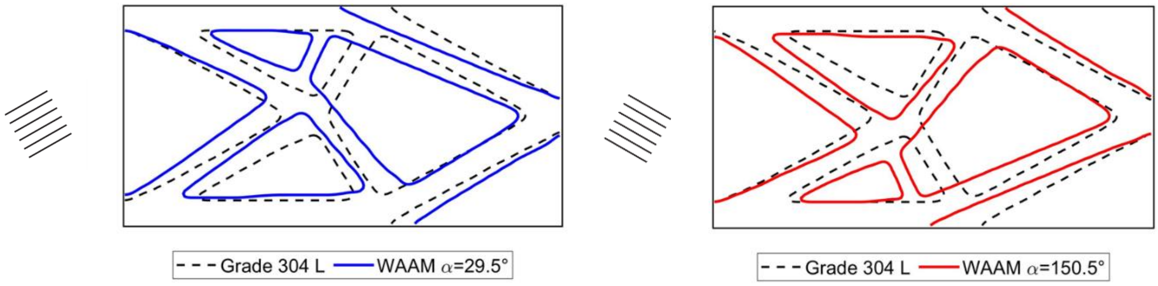

Figure 3. Simultaneous design of the topology and the printing direction for WAAM: optimal results (continuous lines) compared to conventional design with isotropic assumption (dotted line).

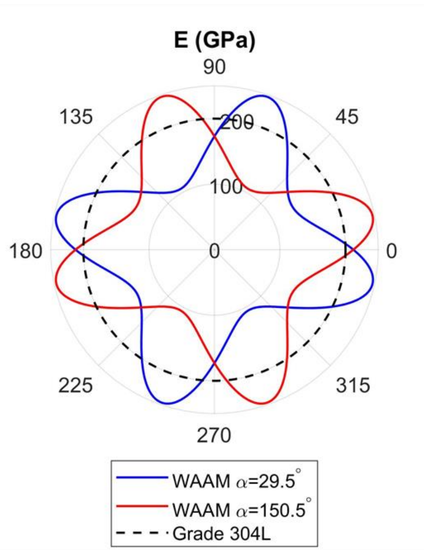

Figure 4. Polar plot of the Young's modulus of WAAM-produced stainless steel for the two optimal printing directions. The angular coordinate identifies the direction along which the modulus is given with respect to x_1. The value for Grade 304 stainless steel is reported for comparison.

4. Conclusions

A tool to design structural elements for Wire-and-Arc Additive Manufacturing has been preliminary discussed. At first, an orthotropic material model has been adopted to process data from experimental tests, thus deriving the compliance tensor of the WAAM alloy in its symmetry axes (the building direction and the transversal one). Hence, a displacement-constrained formulation for the simultaneous design of the topology and of the build direction, which is assumed to remain the same during the whole printing process of the part, has been implemented. The building direction has been embedded in the formulation as an additional degree of freedom with respect to the field of the material density used in standard formulations of topology optimization. A preliminary numerical example has been shown to assess the proposed approach. Optimal layouts have been found in conjunction with non-trivial build directions. The achieved layouts exhibit peculiar features with respect to the conventional result achieved in case of isotropic steel.

Concerning the ongoing developments, the current research is mainly directed towards the handling of three-dimensional models and to the extension of the proposed multi-constrained formulation to meet structural requirements that concern not only serviceability (displacement-based enforcements), but also strength (stress-based enforcements), see e.g. [11].

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