# Multiscale Optimal Design of Grid Systems for High-rise Buildings

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## Introduction

Topology optimization is an appropriate tool to design bracing systems for high-rise buildings [1]. Diagrids and hexagrids are truss structures that employ inclined members instead of vertical columns to carry both vertical and lateral loading. As it is well-known, perimetral grids are an effective way to deal with horizontal forces in tall buildings. Diagrids consist of members that diagonally intersect to generate triangular shapes; hexagrids were inspired by honeycombs, where hexagonal patterns constitute the resisting bulk. Mechanical properties of such kind of grids were extensively investigated and critically review both in the scientific and technical literature [2]. Their optimization is generally performed by using size optimization [1] or homogenization in conjunction with a cantilever beam model of the high-raise building [3].

This paper addresses the optimal design of grid systems in high-rise buildings by means of topology optimization and multiscale analysis of periodic solids. Instead of adopting a multiscale beam model, a 3D box-shaped finite element mesh is used as a discrete design domain to seek for optimal grids whose panels can be regarded as lattice structures. Among the others, [4] computes analytical expressions for the constitutive tensors that describe the elastic behavior of triangular and hexagonal lattices at the macroscopic level (the high-rise building). Such expressions depend on the prescribed reference dimension, cross-section, and material used at the mesoscopic level (the cell).

## Optimization Problem

Along the lines of [5] and [6], a multi-material topology optimization problem is used that employs continuous variables to distribute a discrete set of lattices.

Two types of micro-structure are considered for lattices. One can see in Figure 1(left) an isotropic triangular lattice of a diagrid while in Figure 1(right) the isotropic hexagonal lattice of a hexagrid is presented. The triangular and honeycomb lattices have a six-fold rotational symmetry, so that isotropic constitutive relations are found by means of the multiscale procedure. The constitutive matrices at the macro-scale are:

$$C_d = \frac{3E}{4\sqrt{3}L^3}\begin{bmatrix} 3(AL^2+4I) & AL^2-12I & 0 \\ AL^2-12I & 3(AL^2+4I) & 0 \\ 0 & 0 & AL^2+12I \end{bmatrix} \tag{1}$$

and

$$C_h = \frac{EA}{2\sqrt{3}L(AL^2+12I)}\begin{bmatrix} AL^2+36I & AL^2-12I & 0 \\ AL^2-12I & AL^2+36I & 0 \\ 0 & 0 & 24I \end{bmatrix} \tag{2}$$

for each type of lattice. Here $C_d$ is the constitutive matrix for diagrids while $C_h$ represents the hexagrid.

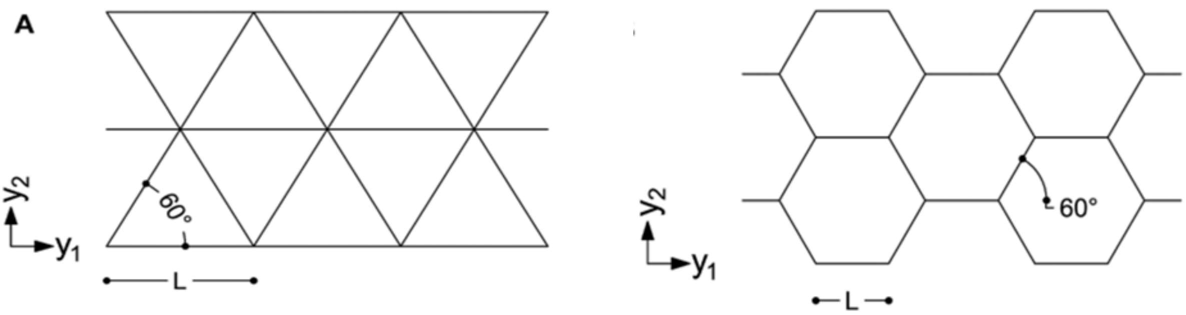


**Figure 1**. An isotropic triangular lattice of a diagrid (left), and the isotropic hexagonal lattice of a hexagrid (right).

Each phase stands for a candidate lattice that has given features in terms of constituent elements and geometrical properties. Diagrids and hexagrids with given reference length L are assumed to be made of elements whose cross-section should be selected within a prescribed set, meaning that the available area A and moment of inertia I have discrete values. A continuous interpolation of the macroscopic stiffness matrix is adopted following an original extension of the SIMP ([5, 6]).

$$C = \rho_0^P\left((1-\rho_1^P)C_1 + \sum_{i=2}^{m-1}(1-\rho_i^P)C_i\prod_{j=1}^{i-1}\rho_j^P + C_m\prod_{j=2}^{m}\rho_{j-1}^P\right) \tag{3}$$

Here in the next, there is no void phase. It is assumed that the whole building envelope is endowed with a structural grid having varying cross-section and shape. A basic lattice that adopts the smallest among the candidate cross-sections provides a minimum stiffness all over the design domain. The goal of the optimization is distributing local increments of the macroscopic stiffness matrix to meet design requirements.

$$C = C_0 + \rho_1^P\left((1-\rho_2^P)(C_1-C_0) + \sum_{i=2}^{m-1}(1-\rho_{i+1}^P)(C_i-C_0)_i\prod_{j=1}^{i-1}\rho_{j+1}^P \right. \\ \left. + (C_m-C_0)\prod_{j=2}^{m}\rho_j^P\right) \tag{4}$$

The above equation handles m+1 cross-sections. $C_0$ is the macroscopic stiffness matrix: (the weakest among the available ones. $C_i$ refers to the i-th cross-section other than the basic one.

Having the aim of using few different sections in the grid the design is straighforward. Here for sake of simplicity, the assumption m = 3 will be used to perform numerical simulations. Writing out the expression in Eq. (4) to handle four cross-sections one has:

$$C = C_0 + \rho_1^P \left((1-\rho_2^P)(C_1 - C_0) + \rho_2^P(1-\rho_3^P)(C_2 - C_0) + \rho_2^P \rho_3^P (C_3 - C_0)\right) \quad (5)$$

Eq.5 is very efficient in penalizing the weights of the stiffness increments, i.e., the optimal values of the continuous variables $\rho$, towards the bounds 0 and 1.

The stated problem searches for the distribution of cross-sections and shapes that minimizes the weight of the grid under enforcements on the stiffness of the building for lateral loads (displacement constraints at the top of the building).

$$\begin{cases} \min\limits_{0 \le x_{p,1},\dots,x_{e,m} \le 1} \mathcal{W} = \sum\limits_{e=1}^{nel} W_e & (a) \\ s.t \left(\sum\limits_{e=1}^{nel} K_e\left(x_{e,1},\dots,x_{e,m}\right)\right) U_1 = F_1 & (b) \\ \left(\sum\limits_{e=1}^{nel} K_e\left(x_{e,1},\dots,x_{e,m}\right)\right) U_2 = F_2 & (c) \\ u_1 \le u_{lim} & (d) \\ u_2 \le u_{lim} & (e) \end{cases} \quad (6)$$

Considering a problem with four candidate cross-sections, the weight of the bars that fall within the e-the finite element reads:

$$W_e = W_{e,0} + x_{e,1}\left[(1 - x_{e,2})(W_{e,1} - W_{e,0}) + x_{e,2}(1 - x_{e,3})(W_{e,2} - W_{e,0}) + x_{e,2}x_{e,3}(W_{e,2} - W_{e,0})\right] \quad (7)$$

In order to improve manufacturability of the achieved layouts, patches are introduced to force the distribution of the same lattice at a given height or within a minimum contiguous area. This also helps in reducing the number of optimization variables used in the optimization.

## Numerical Example

A tall building with square plan (side 48 m) is considered. The height of the building is 207.85 m; a uniform distribution of the horizontal load is assumed along the height of the building with intensity 96 kN/m, acting along the two axes of the plan.

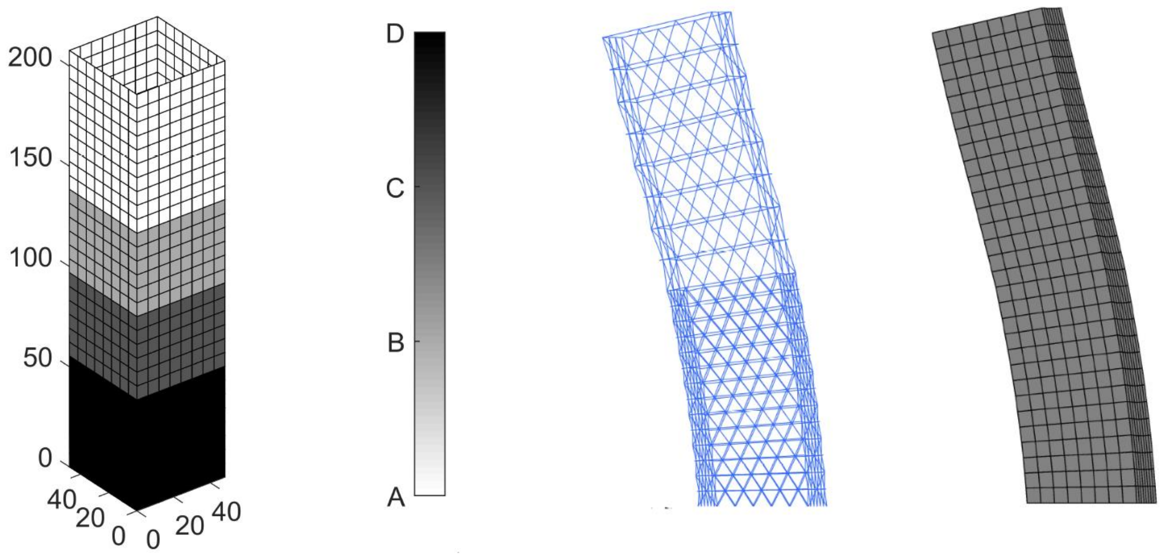


**Figure 2**. A diagrid with varying cross-sections and varying reference dimensions for a tall building with square plan: optimal solution (left); magnified deformed shapes: beam model vs multiscale method (right).

The proposed approach can be used to distribute different skin layouts within the same structural envelope. Two diagrids with reference length L = 16m and L = 8m are considered, defining four candidate equivalent stiffness tensors. The diagrid with reference length L = 16m is made of tubes with circular hollow cross-sections with diameter 558 mm and thickness 12.5mm or 16mm, named diagrid type A and B, respectively; the diagrid with reference length L = 8m consists of tubes having the same cross-section type and equal external diameter but thickness 16mm or 20mm, labelled type C and D respectively. The optimal distribution of lattice structures is sketched in Figure 2. A comparison in terms of (magnified) deformed shapes is presented as well, comparing a beam model and the multiscale one. It is pointed out that the horizontal displacements computed at the top of the grids are in very good agreement. With respect to the achieved optimal solution, the provided stiffness decreases along the height of the building, as expected.

## Conclusion

The conceptual design of grid systems in high-rise buildings has been addressed by combining optimization and multiscale analysis of lattice structures. Macroscopic properties of grids with given cross-section have been retrieved form the literature. Hence, a multi-material optimization problem has been formulated to find the distribution of a prescribed discrete set of candidate cross-sections and shapes such that the structural weight of the grid is minimized under constraints on the lateral displacements of the building. Numerical results have been presented to assess the proposed approach. Current directions of research include testing on more complex geometry of the envelope according to [7].

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