Bayesian approach for efficient identification of highly uncertain structural parameters

M. Ostrowski [1], B. Błachowski [2], G. Mikułowski [3], Ł. Jankowski [4]

Institute of Fundamental Technological Research, Polish Academy of Sciences
[1] mostr@ippt.pan.pl, [2] bblach@ippt.pan.pl, [3] gmikulow@ippt.pan.pl, [4] ljank@ippt.pan.pl

1. Introduction and motivation

This study is devoted to Bayesian approach for parametric identification of a structure equipped with semi-actively lockable joints. Such joints can operate in two different states: unlocked (working as a hinge) or locked (transmitting bending moment). They can be used to provide the effect of modal coupling that allows to precisely control of the mechanical energy flow between vibration modes [1]. The fact that locking/unlocking of the joint removes/adds one rotational degree of freedom (DOF) introduces additional difficulty related to the nonlinear nature of such a reconfigurable system.

Classical methods for modal updating based on mode sensitivity require comparison of modal parameters extracted from measurement data with those obtained numerically. Major disadvantage of this approach is a mode matching problem, because often not all modes are identified during measurement and their order can be inconsistent with the updated model [2, 3]. Moreover, due to mode switching effect numerically calculated modes can change during model updating process. Therefore in this study, a Bayesian approach-based probabilistic framework [4] is proposed that allows to overcome the above mentioned problems.

2. Structure under consideration

Steel structure shown in Figure 1a is equipped with six lockable joints. Beams and lockable joints are connected by screws. Such a connection is characterized by highly uncertain stiffness. Additionally, structural parameters of individual joint-beam connection exhibit significant discrepancies due to inaccuracy of assembly process. These manufacturing errors were clearly visible during measurements by unsymmetrical mode shapes (see: Fig. 3). Hence, each beam-joint is parametrized independently on each other to reproduce these imperfections in the updated model.

In the modal updating procedure only measurements of the structure with joints in locked state are taken into account. The reason is residual friction-driven moment of the joint in unlocked state because the friction parts are still in contact, but with lower value of the clamping force. It causes additional nonlinearities, hence it is more convenient to identify the system for the locked state of the joints. In this case friction force is sufficiently large to avoid slipping of the friction parts in the joint and their relative motion does not take place. Details about the lockable joints can be found in [1, 5].

Finite element (FE) model representing the structure and parametrization of the uncertain beam-joint connections is shown in Figure 1b. A class of FE models $\left(\mathbf{M}, \mathbf{K}(\boldsymbol{\theta})\right) \in \mathcal{C}$ is considered, where $\mathbf{M}$ is constant mass matrix and $\mathbf{K}(\boldsymbol{\theta})$ is stiffness matrix defined as follows:

$$\mathbf{K}(\theta_1, \theta_2, \dots \theta_{N_t}) = \mathbf{K}_0 + \sum_{t=1}^{N_t} \theta_t \mathbf{K}_t \,, \tag{1}$$

where $\mathbf{K}_0$ is stiffness-matrix component related to well-known part of the structure, i.e. to steel profiles, whereas constant matrix-components $\mathbf{K}_t = k_t \mathbf{L}_t^{\mathrm{T}} \mathbf{L}_t$ scaled by the parameters θ_t describe sought stiffnesses of the beam-joint connections. Here this connection is represented by stiffness between two rotational DOFs k_t (see: Fig. 1b, zoomed joint) and $\mathbf{L}_t$ is Boolean matrix responsible for placement of t-th connection. There are 16 parameters describing all beam-joint connections. Steel profiles are represented by FEs based on Euler-Bernoulli beam theory with cubic shape functions. Joints are represented by rigid bodies with defined masses and mass-inertia moments.

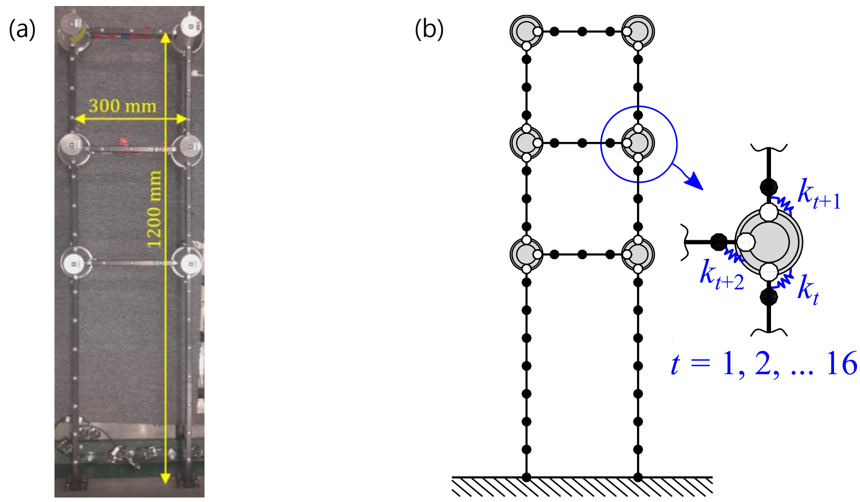


Figure 1. (a) laboratory structure quipped with six lockable joints, (b) mesh of FE model and parametrization of each beam-joint connection

3. Identification of the system parameters with Bayesian approach

In Bayesian approach the most probable values of $\boldsymbol{\theta} = [\theta_1 \quad \theta_2 \quad \cdots \quad \theta_{N_t}]^{\mathrm{T}}$ are sought based on experimental data according to prior gaussian probability density function:

$$J(\boldsymbol{\lambda}, \boldsymbol{\phi}, \boldsymbol{\theta}) = -2 \ln p\big(\boldsymbol{\lambda}, \boldsymbol{\phi}, \boldsymbol{\theta} \big| \boldsymbol{\lambda}_{\text{exp}}, \boldsymbol{\phi}_{\text{exp}}\big) \tag{2}$$

where: $\boldsymbol{\lambda}_{\text{exp}} = [\omega_1^2 \quad \omega_2^2 \quad \cdots \quad \omega_{Nm}^2]^{\mathrm{T}}$ is vector of squares of measured natural frequencies (rad^2/s^2), vector $\boldsymbol{\phi}_{\text{exp}} = \left[\boldsymbol{\phi}_{\text{exp}}^{(1)\,\mathrm{T}} \quad \boldsymbol{\phi}_{\text{exp}}^{(2)\,\mathrm{T}} \quad \cdots \quad \boldsymbol{\phi}_{\text{exp}}^{(N_m)\,\mathrm{T}}\right]^{\mathrm{T}}$ collects measured mode shapes. $\boldsymbol{\lambda}$, $\boldsymbol{\phi}$ and $\boldsymbol{\theta}$ that minimize function J are sought. Since both mathematical model as well as measured data are subjected to certain errors, vectors of random variables $\boldsymbol{\lambda}$ and $\boldsymbol{\phi}$ contain most probable system parameters, whereas $\boldsymbol{\theta}$ most probable parameter values of the FE model. In the Bayesian framework we do not postulate that numerically calculated modes are equal to measured ones so mode matching problem disappears. Iterative procedure of finding optimal values of $\boldsymbol{\lambda}$, $\boldsymbol{\phi}$ and $\boldsymbol{\theta}$ is described in [4].

In order to examine whether parameters are correctly defined the identifiability of the model parameters is checked. Parameters are identifiable at some search space if likelihood $L(\boldsymbol{\theta})$ function has only one maximum over such defined domain [6]. Function $L(\boldsymbol{\theta})$ is given by following equation:

$$
\begin{aligned}
L(\boldsymbol{\theta}) &= K K_\lambda \hat{p}(\mathcal{D}_\lambda | \boldsymbol{\theta}, \mathcal{C}) K_\phi \hat{p}(\mathcal{D}_\phi | \boldsymbol{\theta}, \mathcal{C}) \\
&= K L_\lambda(\boldsymbol{\theta}) L_\phi(\boldsymbol{\theta}),
\end{aligned} \tag{3}
$$

where $\hat{p}(\cdot)$ is prior gaussian probability density function, $\mathcal{D}_\lambda$ is set of measurement data of natural frequencies, $\mathcal{D}_\phi$ is set of measurement data of mode shapes, $L_\lambda(\boldsymbol{\theta})$ is likelihood function corresponding with natural frequencies, whereas $L_\phi(\boldsymbol{\theta})$ for mode shapes, coefficients K, K_λ and K_ϕ are scale factors normalising likelihood functions to one at their maximums. From equation (3) it is evident that identifiability depends not only on chosen parametrization, i.e. class of models $\mathcal{C}$, but also available measurement data contained in $\mathcal{D}_\lambda$ and $\mathcal{D}_\phi$.

4. Results

First, identifiability of vector $\boldsymbol{\theta}$ has been checked. Likelihood function $L(\boldsymbol{\theta})$ has been verified with full-review method for large search space. Hence, tremendous computational effort related to identification of 16 parameters (see: Fig. 1b) has been replaced with reduced two-parameter set: $\tilde{\boldsymbol{\theta}} = [\tilde{\theta}_1 \quad \tilde{\theta}_2]^{\mathrm{T}}$, where $\tilde{\theta}_1$ is related to all vertical beam-joint connections, whereas $\tilde{\theta}_2$ to all horizontal ones. Rotational stiffness $k_t = 10^4$ Nm/rad, $t = 1, 2, \ldots 16$, has been selected. The search space Θ with the following dimensions has been used: $(\tilde{\theta}_1, \tilde{\theta}_2) \in [10^{-1}, 10^2] \times [10^{-2}, 10^2]$.

Only one maximum of likelihood function $L(\tilde{\boldsymbol{\theta}})$ has been found in Θ, so parameters $\tilde{\theta}_1$ and $\tilde{\theta}_2$ are identifiable. Functions $L_\lambda(\tilde{\boldsymbol{\theta}})$, $L_\phi(\tilde{\boldsymbol{\theta}})$ and $L(\tilde{\boldsymbol{\theta}})$ in neighbourhood of optimal values of $\tilde{\boldsymbol{\theta}}$ are shown in Figure 2. Function $L(\tilde{\boldsymbol{\theta}})$ has greater gradients so it provides more precise information about parameters sought than $L_\lambda(\tilde{\boldsymbol{\theta}})$ and $L_\phi(\tilde{\boldsymbol{\theta}})$. It is consequence of greater amount of measurement data.

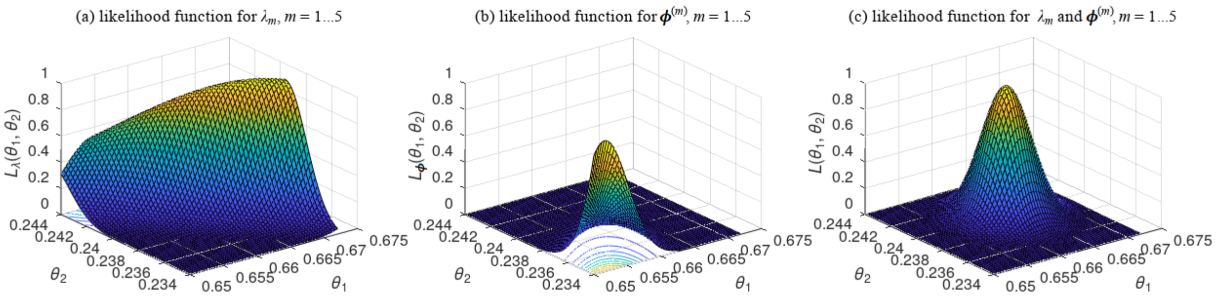


Figure 2. (a-c) likelihood function around neighborhood of optimal parameter values for various measurement data sets of first five modes

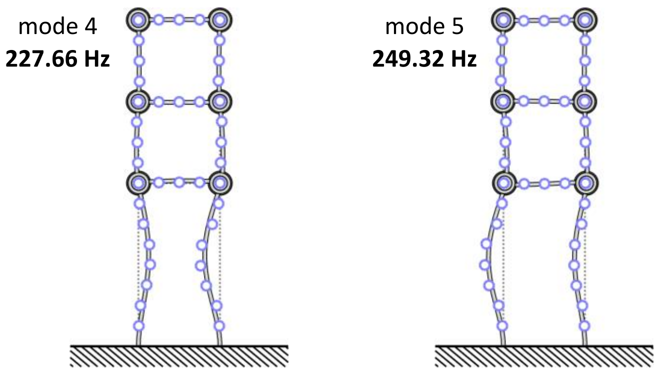
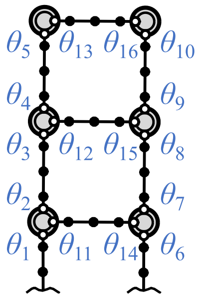


Figure 3. Comparison between mode shapes of updated FE model (gray structure) and measurement data (blue points), natural frequencies in the figure are taken from measurement

Table 1. Final values of: updating parameters, frequency relative error and MAC for updated model

legend:



θ_1	θ_2	θ_3	θ_4	θ_5	θ_6	θ_7	θ_8
0.46	0.90	1.06	1.06	1.08	0.17	0.77	0.28
θ_9	θ_{10}	θ_{11}	θ_{12}	θ_{13}	θ_{14}	θ_{15}	θ_{16}
1.12	1.20	0.32	0.40	0.39	0.24	0.23	0.35

mode [–]	4	5
$\frac{f_m^{\exp} - f_m(\boldsymbol{\theta})}{f_m^{\exp}}$ [%]	-1.67	1.76
MAC [–]	0.89	0.91

Finally, comparison between selected examples of mode shapes identified experimentally and ones calculated numerically using Bayesian approach (eq. (2)) is shown in Figure 3. Initial parameter values for iterative updating procedure were chosen as appropriate parameters $\tilde{\theta}_t$, for which $L(\tilde{\boldsymbol{\theta}})$ achieves maximum, magnified by 1.6. One can see that due to independently parametrized beam-joint connections numerically calculated mode shapes are very well fitted to asymmetric mode shapes identified experimentally. Final values of parameters $\boldsymbol{\theta}$, frequency errors and MAC values are listed in Table 1.

5. Conclusions

Bayesian approach for model updating allows to avoid the mode matching problem. Accurate estimation of the local parameters of the structure subjected to high uncertainties is possible. It includes the higher-order modes that are very sensitive to parametric modifications and can change their order during model updating.

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