Optimal Elastic-Plastic Analysis of Reinforced Concrete Structures with Limited Residual Strain Energy Capacity

M. Movahedi Rad [1], S. Khaleel Ibrahim [2], J. Lógó [3]

[1] Department of Structural and Geotechnical Engineering, Széchenyi István University, H-9026 Győr, Hungary, majidmr@sze.hu

[2] Department of Structural and Geotechnical Engineering, Széchenyi István University, H-9026 Győr, Hungary, sarahibrahim8391@gmail.com

[3] Department of Structural Mechanics, Department of Highway and Railway Engineering, Budapest University of Technology and Economics, H-1111, Budapest, Hungary, logo.janos@emk.bme.hu

1. Introduction

One of the most successful applications of the variational formulation in the incremental plasticity theory is the theory of limit analysis. The basic ideas of the principles of limit analysis were first recognized and applied to the steel beams by Kazinczy [1]. The fundamental problem of limit analysis is to determine the plastic limit load multiplier and the stresses, strain rates and velocities at the plastic limit state of the body. However, at the application of the plastic analysis and design methods the control of the plastic behaviour of the structures is an important requirement. Since the limit analysis provides no information about the magnitude of the plastic deformations and residual displacements accumulated before the adaptation of the structure, therefore for their determination several bounding theorems and approximate methods have been proposed. Among others Kaliszky and Lógó [2] suggested that the complementary strain energy of the residual forces could be considered an overall measure of the plastic performance of structures and the plastic deformations should be controlled by introducing a bound for magnitude of this energy.

In this research, optimal elastic-plastic analysis and design methods of reinforced concrete (RC) structures are presented using limited plastic deformation. First, a concrete plastic damage (CPD) constitutive model is developed and applied to calibrate a numerical model. Then, different objective functions were considered to optimize the volume of the steel used to reinforce the structures and plastic loading. The plastic deformations are controlled by using constraints on the complementary strain energy of the residual internal forces of the steel bars. Applying different optimization problems showed that the complementary strain energy of the residual forces has a significant effect and can be considered as a constraint on the plastic behavior of the RC structures.

2. Methods

Let us assume that the steel bar elements has been defined by the concept of elastic-plastic analysis and design methods. Therefore by applying load P_0 plastic forces Q^p will appear in the structure. When the load is reduced under unloading elastic deformations occur and then the elastic internal forces $-Q^e$ will take place in the structure. Accordingly after unloading the residual internal forces remain in the structure.

$$Q^r = Q^p - Q^e \tag{1}$$

Where

$$Q^e = F^{-1}GK^{-1}P_0 \tag{2}$$

here K is the stiffness matrix; F is the flexibility matrix and G denotes the geometrical matrix. Therefore the complementary strain energy for positive-definite function, can be determined from the residual forces.

$$C_r = \frac{1}{2}\sum_{i=1}^{n}\frac{1}{S_i}\int_0^{l_i}\left(Q_i^p(s) - Q_i^e(s)\right)^2 ds \geq 0 \tag{3}$$

here $Q_i^p(s)$ and $Q_i^e(s)$ are the functions of plastic and elastic internal forces; S_i expresses tensile stiffnesses for truss elements respectively.

A proper computational method proposed that the complementary strain energy of the internal residual forces could be defined as a general measure of the plastic performance of the structures and the residual deformations are constrained by introducing a limit for the value of this energy:

$$\frac{1}{2}\sum_{i=1}^{n}\frac{1}{S_i}\int_0^{l_i}\left(Q_i^p(s) - Q_i^e(s)\right)^2 ds - C_{r0} \leq 0 \tag{4}$$

where C_{r0} is an allowable strain energy value for C_r.

For steel bar elements the complementary strain energy of the residual forces can be considered as follow:

$$C_p = \frac{1}{2E}\sum_{k=1}^{n}\frac{l_k}{A_k}(S_k^r)^2 \tag{3}$$

Here A_k is the cross-section area the bar elements $l_k, (k = 1, 2, \dots, n)$ denotes the length of the bars, S_k^r: the residual normal force of the bar members, E: the Young's modulus.

3. Conclusion

In this study elastic-plastic analysis of RC structures with limited plastic displacements introduced. For this goal plastic limit theorem is developed and applied to the structures. Additionally to control the plastic behavior of the structure the complementary strain energy of the residual forces are constrained. A concrete plastic damage (CPD) constitutive model is applied to calibrate a numerical model, then different nonlinear optimization problems were carried out concerning the reinforcement steel volume ratio and plastic loading. The numerical results shows that the bound of the complementary strain energy of the residual forces can influence significantly the magnitude of the limit load multiplier.

References

[1] **G. Kazinczy.** (1914). *Experiments with Clamped Beams*. Betonszemle 2, No. 4,5,6 , 68-71, 83-87, 101-104.

[2] **S. Kaliszky, J. Lógó.** (1997). *Optimal Plastic Limit and Shakedown Design of Bar Structures with Constraints on Plastic Deformation*, Engineering Structures, 19 (1), pp. 19-27.